# Mathematical Model and Method for Compilation of Urban Rail Transit Train Operation Diagram

**Luo Yuan**

Sichuan Changjiang Vocational College, Chengdu, 610009, China

**Keywords:** Urban rail transit; Train operation diagram compilation; Mathematical model

**Abstract:** The collaborative compilation of train schedules plays a core role in the networked operation of urban rail transit, and its quality directly affects the service and operational efficiency of the urban rail system. Faced with the increasingly complex urban rail network, how to scientifically and efficiently develop train schedules to meet the dynamically changing passenger demand has become a key issue that urgently needs to be overcome. The train schedule is closely linked to passenger flow demand: it is not only a key reference for passengers to plan their itinerary and choose their mode of travel, but also provides detailed information on train operation time and stopping stations; At the same time, fluctuations in passenger demand also have a negative impact on the development of train schedules, requiring them to have sufficient flexibility to adjust and adapt to changes in passenger flow during different periods and directions. When the capacity of the train schedule is highly compatible with passenger demand, the urban rail system can not only effectively meet the travel needs of passengers, improve passenger satisfaction, but also maximize the utilization of transportation potential, reduce operating costs, and thereby enhance operational efficiency. In view of this, this article explores in detail the mathematical modeling and practical methods for the compilation of urban rail train operation diagrams.

## 1. Introduction

With the rapid development of the Chinese economy and the acceleration of urbanization, urban rail transit has become an important component of the modern urban transportation system due to its high efficiency, environmental protection, and punctuality [1]. In recent years, the rapid advancement of China's urban rail transit network has not only greatly improved the level of transportation interconnection within and between cities, but also significantly enhanced the accessibility and convenience of passenger travel [2]. This transformation not only reflects the transportation revolution brought about by technological progress, but also profoundly embodies the urgent demand of society for efficient and green modes of transportation [3]. In this context, passengers' travel needs are becoming increasingly diversified, expanding from basic commuting needs to leisure, tourism, and other aspects, which puts higher demands on the operation management and service quality of urban rail transit systems [4]. As the core element of urban rail transit transportation organization, the scientificity and rationality of the train schedule directly affect the transportation efficiency, service quality, and emergency response capability of the entire system.

The train schedule provides a detailed plan of the arrival, departure, or passage times of trains in various sections and stations, as well as the interrelationships between trains. It is the foundation of train command, train scheduling, passenger information services, and other work [5]. With the continuous expansion and complexity of urban rail transit networks, how to develop train schedules that meet operational needs and efficiently respond to emergencies has become a key issue that urgently needs to be addressed [6]. Traditional train operation diagram compilation relies heavily on manual experience, which is not only time-consuming and laborious, but also difficult to ensure the optimal solution. With the rapid development of information technology, automated and intelligent train operation diagram preparation systems have emerged, greatly improving the efficiency and accuracy of preparation [7]. However, most of the current computer mapping systems in China are still limited to the application of a single line. For the complex and ever-changing urban rail transit network, especially in terms of collaborative operation between multiple lines and seamless connection with other modes of transportation such as buses, taxis, airlines, etc., the functions of the existing systems are inadequate [8].

In addition, in the face of emergency situations such as natural disasters and equipment failures, the existing network operation diagram preparation system often lacks the ability to respond quickly and adjust flexibly, making it difficult to effectively respond to changes in transportation demand in emergency situations, which affects the overall resilience and passenger experience of the rail transit system. Therefore, in-depth exploration of mathematical modeling and practical methods for urban rail transit train operation diagrams, and the development of a train operation diagram preparation system that can adapt to networked operations, highly collaborative, and intelligent decision-making, are of great significance for improving the overall operational efficiency of the rail transit system, enhancing service flexibility, and optimizing resource allocation. This article aims to explore in detail the mathematical modeling and practical methods for the compilation of urban rail train operation diagrams. By analyzing the limitations of existing systems and combining them with the actual needs of urban rail transit network operation, innovative compilation strategies and technical paths are explored.

## 2. Limitations of Existing Systems

As a key link in the operation and management of urban rail transit, the complexity and limitations of the urban rail train operation diagram compilation system are particularly prominent under the interweaving of multidimensional factors [9]. The preparation of the running diagram for each route not only needs to cover the selection of routing modes (such as single routing, large and small routing, et al.), but also needs to be closely integrated with the changing trends of passenger flow and the rigid constraints of equipment parameters. These factors together constitute the basic framework for the preparation of the running diagram [10]. Figure 1 shows the distribution of factors affecting the formulation of train routing plans for urban rail transit. However, as the perspective expands from a single line to the entire network level, the complexity of the problem increases exponentially. The interaction between various lines within the network is complex and intricate, while external factors such as urban planning, transportation policies, and weather changes affect it. This intertwined relationship requires a high degree of global perspective and overall planning ability in the compilation work.

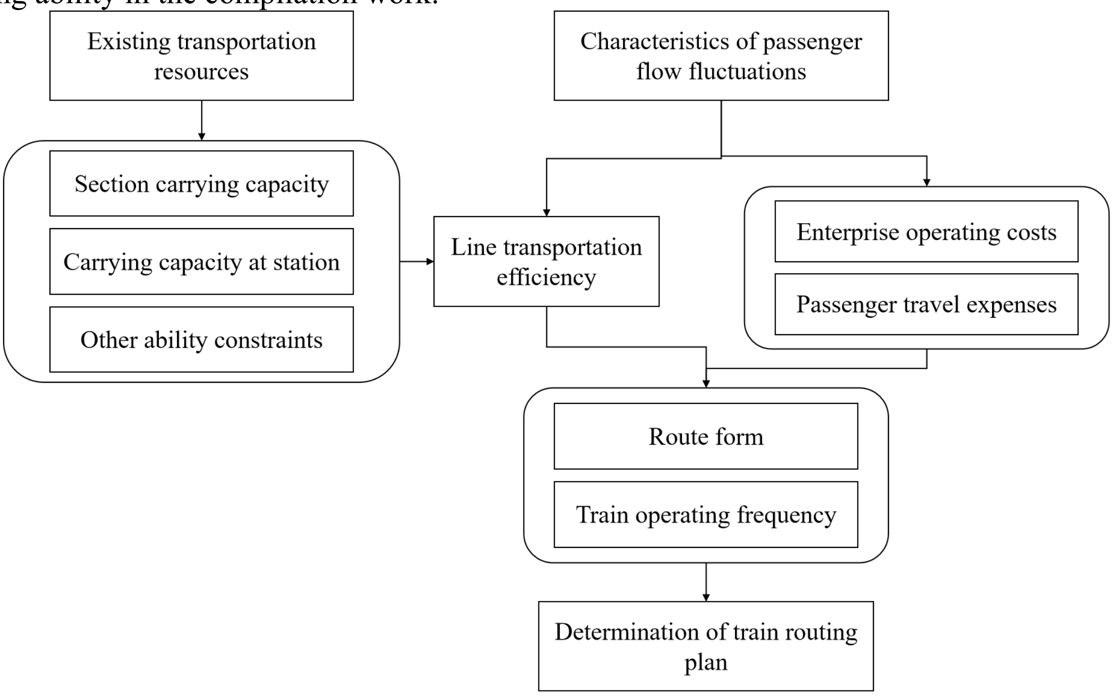


Figure 1 Distribution of influencing factors

As the direct target of urban rail transit services, the dynamic characteristics and diversity of passenger flow pose severe challenges to the compilation of operation diagrams. With the acceleration of urbanization and the diversification of residents' travel patterns, the demand for passenger flow presents more significant spatiotemporal fluctuations and uncertainties. The traditional traffic organization based on a fixed timetable and the mode of relying on manual experience to adjust the timetable are no longer effective in dealing with this dynamic change, especially when there is a significant difference in passenger flow between peak and off peak periods, which is even more inadequate. The problem of transfer and connection has further intensified the difficulty of compiling the operation diagram. In urban rail transit networks, transfer stations serve as hubs connecting different lines, and accurate matching of train arrival and departure times is crucial for improving overall transportation efficiency. Figure 2 shows a schematic diagram of the transfer network structure of urban rail transit.

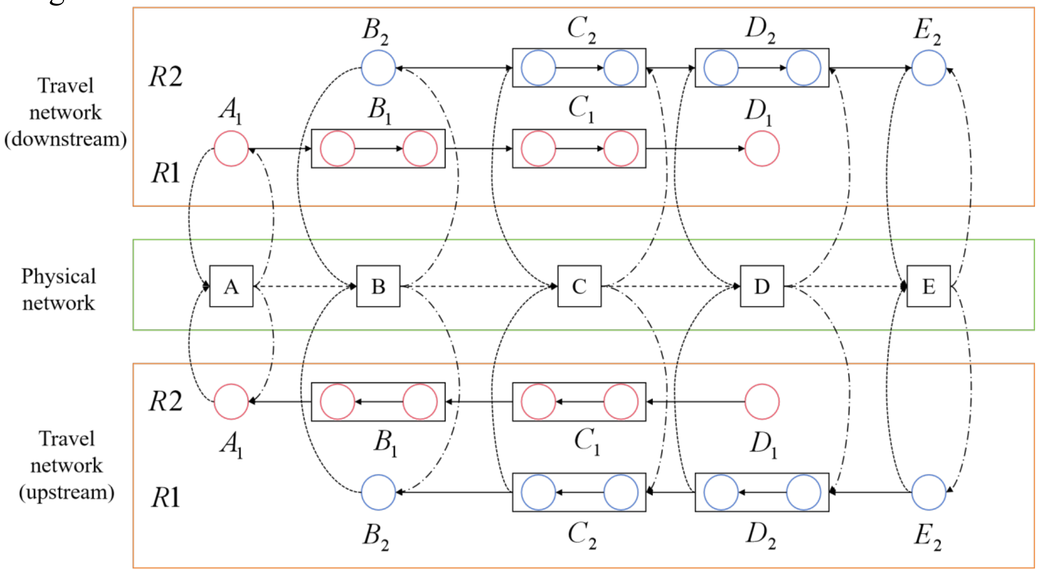


Figure 2 Schematic diagram of transfer network structure

However, due to the interdependence and mutual constraints of train operation plans on various lines, any adjustment of a line may affect the entire network, making the coordination of transfer connections an extremely complex and delicate task. Especially in the face of different passenger flow characteristics and significant temporal changes on different routes, how to ensure smooth and unobstructed transfers has become an urgent technical challenge to be solved. In addition, the connection between some stations and other modes of transportation such as railways and aviation has made passenger flow changes more complex and varied, with frequent irregular fluctuations. This not only requires the operation chart preparation system to have high flexibility and adaptability, but also to be able to integrate advanced technologies such as big data analysis and artificial intelligence (AI) prediction in real time to achieve accurate grasp and rapid response to passenger flow changes.

## 3. Mathematical Modeling and Methods

### 3.1. Mathematical Modeling

The core of compiling the urban rail train schedule lies in efficiently and accurately matching passenger demand and transportation capacity to optimize the train operation plan. However, the commonly used maximum cross-sectional transport capacity matching method in existing practice has limitations due to its simplified processing method and neglect of diverse passenger flow characteristics, resulting in the quality of the operation plan not fully realizing its potential. Faced with this challenge, exploring the optimization of train routing schemes based on passenger flow matching has become the key to improving transportation efficiency and service quality. This problem is essentially a complex nonlinear integer programming problem that requires consideration of multiple constraints, such as train capacity, station stopping time, passenger transfer demand, etc., in order to find the optimal train operation pairs, departure intervals, and routing combinations.

Although traditional genetic algorithms (GA) have been applied to such problems, their search efficiency and ability to obtain global optimal solutions still need to be improved. The immune cloning algorithm, with its unique mechanism of simulating the biological immune system, provides a new solution to this problem. By introducing antigen antibody simulation, affinity evaluation mechanism, and memory storage, this algorithm can more effectively explore the solution space and accelerate the convergence process towards the global optimal solution. This article applies this algorithm to the optimization of train routing schemes in urban rail transit. By finely designing algorithm components, such as analogizing the objective function as an antigen, treating feasible solutions as antibodies, and defining the degree of match between solutions and objectives as affinity, memory cells record and iteratively update the optimal solution set, thereby achieving refined optimization of train operation schemes and providing more scientific and efficient decision support for urban rail transit operations.

### 3.2. Algorithm Design

The full turnover running time of the train is defined as the time required for the train to depart from the starting station of the operating line, return to the starting station after passing through the terminal station, and complete the entire process of preparing for another departure. This time element occupies a central position in the train schedule and has a profound impact on the formulation of train operation plans, routing plans, and vehicle utilization plans. It consists of several key components: interval running time, stopping time, and turnaround time.

$$T_Z = \sum T_i + \sum T_j + 2t_Z \tag{1}$$

In the formula, $\sum T_i$ represents the total running time of the train in all sections; $\sum T_j$ represents the cumulative stopping time of trains at all stations; And $t_Z$ is the time required for a single turnaround.

The design of train routing schemes for urban rail transit mainly includes two key aspects: firstly, the determination of the routing form, which is usually carefully selected from a set of alternative routes; The second is the setting of the frequency of train operation for each route, which requires a detailed evaluation of its feasibility variation and precise selection within $\left[f_{l_i,\min}, f_{l_i,\max}\right]$. This article proposes a novel antibody coding method to address this issue.

$$F = \left(f_1, f_2, \cdots, f_{N_r}\right) \tag{2}$$

In the formula, $f_i$ represents the frequency of train operation for route $i$. If route $i$ is not selected, the corresponding frequency value $f_i = 0$. The length of the antibody is equal to the number of alternative pathways $N_r$.

Passengers tend to choose the option with the highest travel utility when choosing their travel route, which can essentially be seen as a probability based selection problem. This probability can be specifically explained as the proportion of passengers who choose the $w$ path within interval $(r,s)$, which can be expressed mathematically as:

$$p_w^{rs} = \Pr\left(U_w^{rs} \geq U_n^{rs}\right) \tag{3}$$

In the formula, $p_w^{rs}$ represents the probability of passengers choosing path $w$ within interval $(r,s)$; $\Pr\left(U_w^{rs} \geq U_n^{rs}\right)$ represents the probability that the random utility of the $w$ path within interval $(r,s)$ is greater than or equal to the random utility of any other path $n$.

The optimization of urban rail transit routing plans aims to optimize the routing form and the frequency of train operation on each route based on the principle of passenger flow matching. Therefore, in the process of antibody mutation, both aspects of mutation operation must be taken into account, and both are indispensable. In view of this, this article innovatively integrates simulated annealing algorithm into antibody mutation operation. Specifically, the process first randomly swaps the elements on each column in the encoding of each antibody, ensuring that the total number of pathways remains unchanged, thereby generating mutated new antibodies. The calculation of the mutation probability $p$ is based on the following formula:

$$p = \max\left( p_{\max} \cdot \left(1 - \frac{m}{M}\right), p_{\min} \right) \tag{4}$$

In the formula, $m$ represents the current number of evolutions, while $M$ is the preset maximum number of evolutions. $p_{\max}, p_{\min}$ represents the maximum and minimum values of mutation probability, where $M, p_{\max}, p_{\min}$ is a preset fixed value.

## 4. Conclusions

The collaborative compilation of train operation diagrams for urban rail transit networks marks an important step towards intelligent management of urban rail transit operations. This innovative practice not only significantly improves the efficiency and accuracy of train operation diagram compilation in the railway network, but also greatly enhances the coordination and overall planning of urban rail network transportation organization. By integrating and optimizing the train operation plans between various lines, this strategy effectively taps into and unleashes the transportation potential of the entire urban rail network system, promoting the maximization of comprehensive transportation benefits. The application of immune cloning algorithm in the optimization and compilation of urban rail transit train schedules explored in this article is an innovative attempt at traditional compilation methods. This algorithm provides strong technical support for optimizing train operation diagrams with its unique search mechanism and efficient problem-solving ability. Looking ahead to the future, with the flourishing development of big data and AI technology, we can further expand the boundaries of this research field. For example, exploring the deep integration of real-time passenger flow prediction technology and dynamic train operation chart adjustment strategies to achieve real-time response and dynamic optimization of train operation charts to changes in passenger flow. This will not only enhance the flexibility of train operation and the convenience of passenger travel, but also open up new paths for the intelligent and efficient development of urban rail transit systems.

## Acknowledgments

1) Issued in 202308, issued by the National Vocational Education Planning Leading Group Office, the topic is "Promoting Teaching with Competitions and Integrating Competitions and Teaching" to explore the talent training model, and the topic number is 2022QJZ027.

2) Issued in 202304, issued by the General Office of the National Education and Scientific Research Leading Group, the subject is "Professional Course Construction Based on Job Competency Analysis - Taking Urban Rail Transit Passenger Organization as an Example", subject number 2021HER015.

## References

[1] Seo T, Wada K, Fukuda D. Fundamental diagram of urban rail transit considering train–passenger interaction [J]. Transportation, 2023, 50(4): 1399-1424.

[2] Chen Y, Li T, Sun Y, et al. Dynamic data-driven computation method for the number of waiting passengers and waiting time in the urban rail transit network[J]. IET Intelligent Transport Systems, 2023, 17(1): 165-179.

[3] Lian D, Mo P, D'Ariano A, et al. Energy-saving time allocation strategy with uncertain dwell times in urban rail transit: Two-stage stochastic model and nested dynamic programming framework [J]. European Journal of Operational Research, 2024, 317(1): 219-242.

[4] Zhao Q, Tang J, Zhang X. Collaborative passenger flow control for an urban rail transit network [J]. Computer-Aided Civil and Infrastructure Engineering, 2024, 39(1): 63-85.

[5] Luo Q, Lin B, Lyu Y, et al. Spatiotemporal path inference model for urban rail transit passengers based on travel time data[J]. IET Intelligent Transport Systems, 2023, 17(7): 1395-1414.

[6] Yu Q, Zhang Y, Guo J, et al. A multiple spatio-temporal features fusion approach for short-term passenger flow forecasting in urban rail transit[J]. IET Intelligent Transport Systems, 2023, 17(9): 1729-1741.

[7] Zhu F, Yang Z, Zhao Z, et al. Two-stage synthetic optimization of supercapacitor-based energy storage systems, traction power parameters and train operation in urban rail transit[J]. IEEE Transactions on Vehicular Technology, 2021, 70(9): 8590-8605.

[8] Wang D, D'Ariano A, Zhao J, et al. Integrated rolling stock deadhead routing and timetabling in urban rail transit lines[J]. European Journal of Operational Research, 2022, 298(2): 526-559.

[9] Mo X, Cao C, Li M, et al. Predicting the impact of disruptions to urban rail transit systems[J]. ACM Transactions on Sensor Networks, 2022, 19(1): 1-17.

[10] Deng L, Chen M, Duan K, et al. Integrated energy-efficient optimization for urban rail transit timetable [J]. IET Intelligent Transport Systems, 2023, 17(9): 1819-1834.